

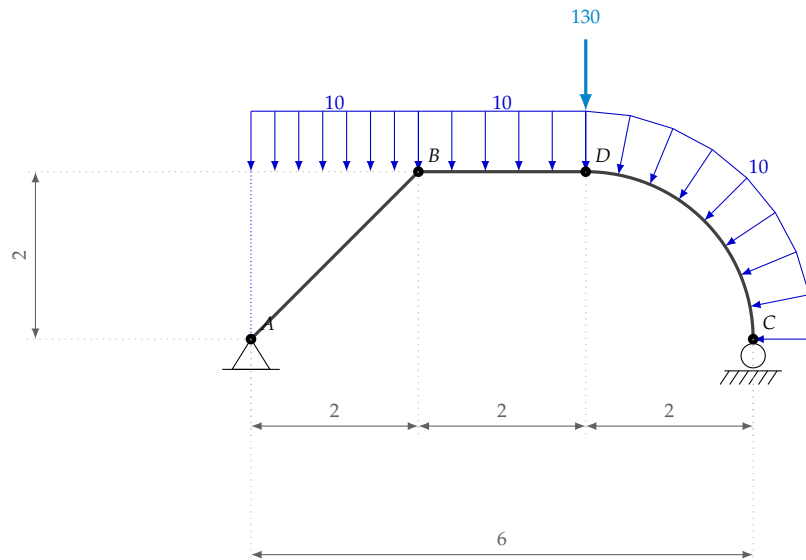
Método de cortes en arcos (leyes en θ)

PÓRTICO

EMI · solución paso a paso

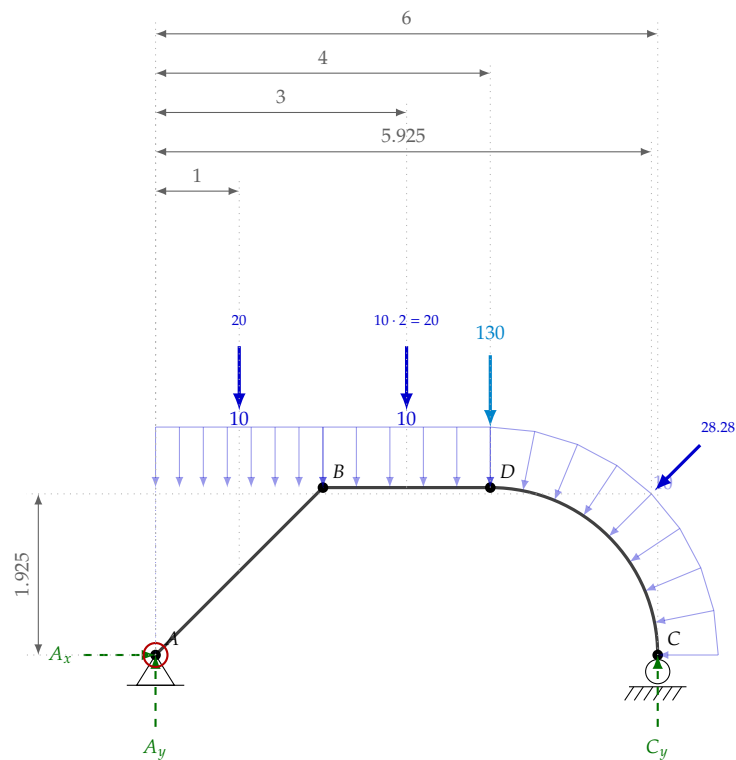
Unidades de trabajo: L [m] F [kgf] M [kgf m] q [kgf/m] δ [mm] E [kgf/m²] A [m²] I [m⁴]

1. Datos y clasificación estática



Geometría, apoyos y cargas de la estructura analizada.

2. Reacciones en los apoyos



$$(\rightarrow +) \sum F_x = 0 \Rightarrow A_x - 20 = 0 \quad \dots \text{(I)}$$

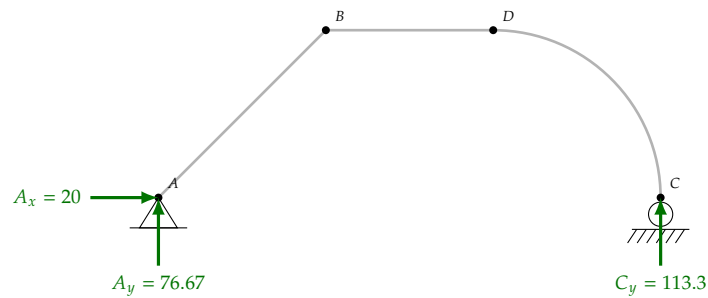
$$\begin{aligned} (\uparrow +) \sum F_y = 0 &\Rightarrow A_y + C_y - 130 - 20 - 20 - 20 = 0 \\ &\Rightarrow A_y + C_y - 190 = 0 \quad \dots \text{(II)} \end{aligned}$$

$$\begin{aligned} (\cup +) \sum M_A = 0 &\Rightarrow 6 C_y - 520 - 20 - 80 - 60 = 0 \\ &\Rightarrow 6 C_y - 680 = 0 \quad \dots \text{(III)} \end{aligned}$$

$$\Rightarrow A_x = 20 \text{ kgf}$$

$$\Rightarrow A_y = 76.67 \text{ kgf}$$

$$\Rightarrow C_y = 113.3 \text{ kgf}$$



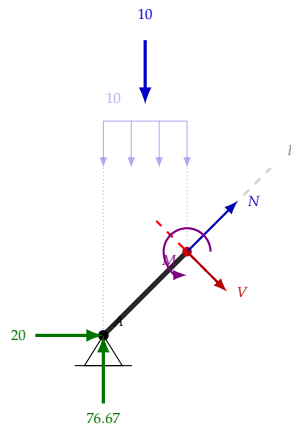
Reacciones en los apoyos (kgf, kgfm).

3. Esfuerzos internos por barras (método de cortes)

Barra A-B $0 \leq x \leq 2.828 \text{ (m)}$

$$M(x) = -\sum M_A + x (\mathbf{t} \times \Sigma \mathbf{F}), \quad N(x) = -\Sigma \mathbf{F} \cdot \mathbf{t}, \quad V(x) = \Sigma \mathbf{F} \cdot \mathbf{n}$$

Tramo 1: $0 \leq x \leq 2.828$



$$\Delta F_y^{(1)} = \int_0^x (-7.071) dt = -7.071 x$$

$$\Delta M_A^{(1)} = \int_0^x (-5 t) dt = -2.5 x^2$$

$$\begin{aligned} \Sigma F_x(x) &= A_x + \Delta F_x^{(1)} \\ &= 20 + 0 \\ &= 20 \end{aligned}$$

$$\begin{aligned} \Sigma F_y(x) &= A_y + \Delta F_y^{(1)} \\ &= 76.67 - 7.071 x \end{aligned}$$

$$\begin{aligned} N_{AB}(x) &= -\Sigma \mathbf{F} \cdot \mathbf{t} \\ &= -68.35 + 5 x \end{aligned}$$

$$\begin{aligned} V_{AB}(x) &= \Sigma \mathbf{F} \cdot \mathbf{n} \\ &= 40.07 - 5 x \end{aligned}$$

$$M_{w_1}(x) = -\Delta M_A^{(1)} + x (\mathbf{t} \times \Delta \mathbf{F}^{(1)}) = -2.5 x^2$$

$$\begin{aligned}
 M_{AB}(x) &= A_y x - A_x (y - 4) + M_{w1}(x) \\
 &= 76.67 (0.7071 x) - 20 (0.7071 x) - 2.5 x^2 \\
 &= 40.07 x - 2.5 x^2
 \end{aligned}$$

$$\begin{aligned}
 x = 0 (A) &\rightarrow \begin{cases} N(0) = -68.35 \text{ kgf} \\ V(0) = 40.07 \text{ kgf} \\ M(0) = 0 \text{ kgf m} \end{cases} \\
 x = 2.828 &\rightarrow \begin{cases} N(2.828) = -54.21 \text{ kgf} \\ V(2.828) = 25.93 \text{ kgf} \\ M(2.828) = 93.33 \text{ kgf m} \end{cases}
 \end{aligned}$$

Barra D-C $0 \leq \theta \leq \frac{\pi}{2}$ (rad)

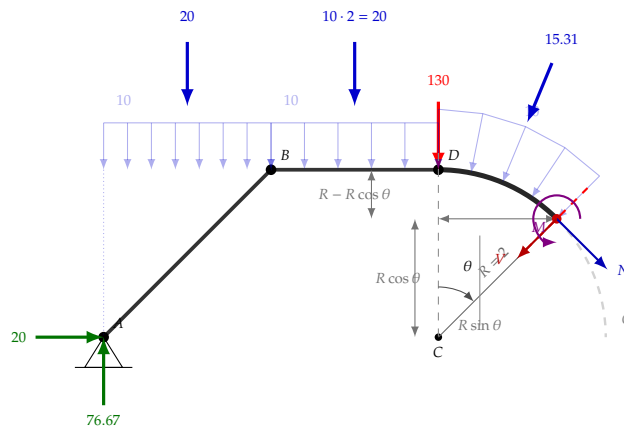
$$x(\theta) = 4 + 2 \sin \theta, \quad y(\theta) = 4 + 2 \cos \theta, \quad ds = R d\theta$$

$$\begin{aligned}
 M(\theta) &= -\Sigma M_C + (x - x_C) \Sigma F_y - (y - y_C) \Sigma F_x \\
 &= -\Sigma M_C + (2 \sin \theta) \Sigma F_y - (2 \cos \theta) \Sigma F_x
 \end{aligned}$$

$$N(\theta) = -\Sigma \mathbf{F} \cdot \mathbf{t} = -[(\cos \theta) \Sigma F_x + (-\sin \theta) \Sigma F_y]$$

$$V(\theta) = \Sigma \mathbf{F} \cdot \mathbf{n} = (\sin \theta) \Sigma F_x + (\cos \theta) \Sigma F_y$$

Tramo 1: $0 \leq \theta \leq \frac{\pi}{2}$ (0° a 90°)



$$\Delta F_x^{(1)} = \int_0^\theta (-20 \sin t) dt = -20 + 20 \cos \theta$$

$$\Delta F_y^{(1)} = \int_0^\theta (-20 \cos t) dt = -20 \sin \theta$$

$$\begin{aligned}
 \Sigma F_x(\theta) &= A_x + \Delta F_x^{(1)} \\
 &= 20 + (-20 + 20 \cos \theta) \\
 &= 20 \cos \theta
 \end{aligned}$$

$$\begin{aligned}
 \Sigma F_y(\theta) &= A_y - 130 - 20 - 20 + \Delta F_y^{(1)} \\
 &= 76.67 - 130 - 20 - 20 - 20 \sin \theta \\
 &= -93.33 - 20 \sin \theta
 \end{aligned}$$

$$\begin{aligned}
 N_{DC}(\theta) &= -[(\cos \theta) \Sigma F_x + (-\sin \theta) \Sigma F_y] \\
 &= -20 - 93.33 \sin \theta
 \end{aligned}$$

$$\begin{aligned}
 V_{DC}(\theta) &= (\sin \theta) \Sigma F_x + (\cos \theta) \Sigma F_y \\
 &= -93.33 \cos \theta
 \end{aligned}$$

$$M_{w1}(\theta) = (x - x_C) \Delta F_y^{(1)} - (y - y_C) \Delta F_x^{(1)} - \Delta M_C^{(1)} = -40 + 40 \cos \theta$$

$$\begin{aligned}
 M_{DC}(\theta) &= A_y x - A_x (y - 4) - 130 (x - 4) - 20 (x - 1) - 20 (x - 3) + M_{w1}(\theta) \\
 &= 76.67 (4 + 2 \sin \theta) - 20 (2 \cos \theta) - 130 (2 \sin \theta) \\
 &\quad - 20 (3 + 2 \sin \theta) - 20 (1 + 2 \sin \theta) + (-40 + 40 \cos \theta) \\
 &= 186.7 - 186.7 \sin \theta
 \end{aligned}$$

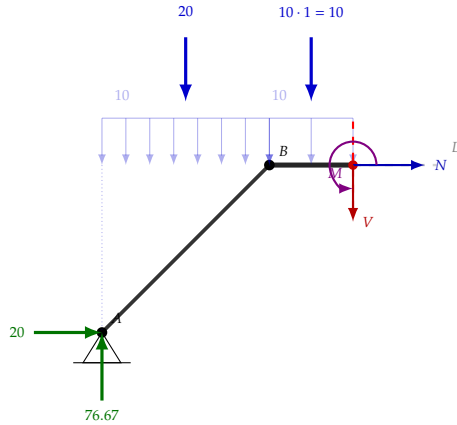
Tabulando θ en el tramo:

θ (rad)	θ (grados)	N (kgf)	V (kgf)	M (kgf m)	Punto
0	0°	-20	-93.33	186.7	D
$\pi/6$	30°	-66.67	-80.83	93.33	
$\pi/3$	60°	-100.8	-46.67	25.01	
$\pi/2$	90°	-113.3	0	0	C

Barra $B-D$ $0 \leq x \leq 2$ (m)

$$M(x) = -\Sigma M_B + x(\mathbf{t} \times \Sigma \mathbf{F}), \quad N(x) = -\Sigma \mathbf{F} \cdot \mathbf{t}, \quad V(x) = \Sigma \mathbf{F} \cdot \mathbf{n}$$

Tramo 1: $0 \leq x \leq 2$



$$\Delta F_y^{(1)} = \int_0^x (-10) dt = -10x$$

$$\Delta M_B^{(1)} = \int_0^x (-10t) dt = -5x^2$$

$$\begin{aligned} \Sigma F_x(x) &= A_x + \Delta F_x^{(1)} \\ &= 20 + 0 \\ &= 20 \end{aligned}$$

$$\begin{aligned} \Sigma F_y(x) &= A_y - 20 + \Delta F_y^{(1)} \\ &= 76.67 - 20 - 10x \\ &= 56.67 - 10x \end{aligned}$$

$$\begin{aligned} N_{BD}(x) &= -\Sigma \mathbf{F} \cdot \mathbf{t} \\ &= -20 \end{aligned}$$

$$\begin{aligned} V_{BD}(x) &= \Sigma \mathbf{F} \cdot \mathbf{n} \\ &= 56.67 - 10x \end{aligned}$$

$$M_{w1}(x) = -\Delta M_B^{(1)} + x(\mathbf{t} \times \Delta \mathbf{F}^{(1)}) = -5x^2$$

$$\begin{aligned} M_{BD}(x) &= A_y x - A_x (y - 4) - 20(x - 1) + M_{w1}(x) \\ &= 76.67(2 + x) - 20(2) - 20(1 + x) - 5x^2 \\ &= 93.33 + 56.67x - 5x^2 \end{aligned}$$

$$\begin{aligned} x = 0 (B) &\rightarrow \begin{cases} N(0) = -20 \text{ kgf} \\ V(0) = 56.67 \text{ kgf} \\ M(0) = 93.33 \text{ kgf m} \end{cases} \\ x = 2 (D) &\rightarrow \begin{cases} N(2) = -20 \text{ kgf} \\ V(2) = 36.67 \text{ kgf} \\ M(2) = 186.7 \text{ kgf m} \end{cases} \end{aligned}$$

Diagramas de fuerzas internas

Esfuerzo axial N , fuerza cortante V y momento flector M

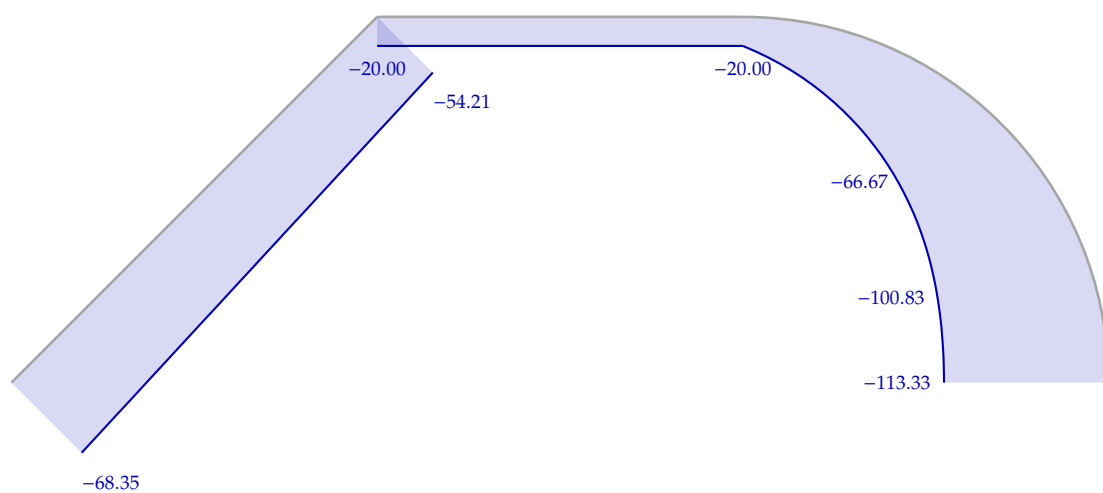


Diagrama de fuerza axial (N)

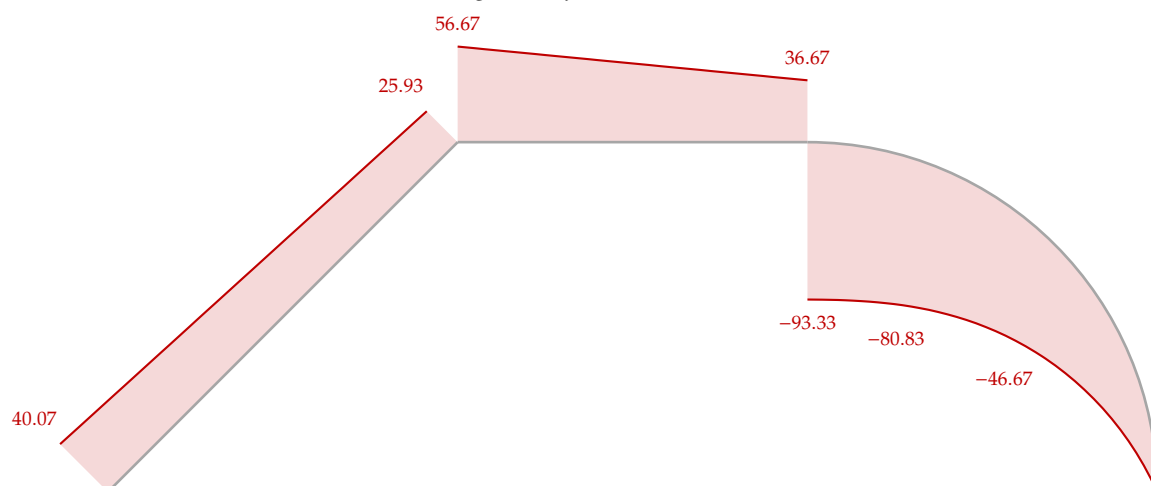


Diagrama de fuerza cortante (V)

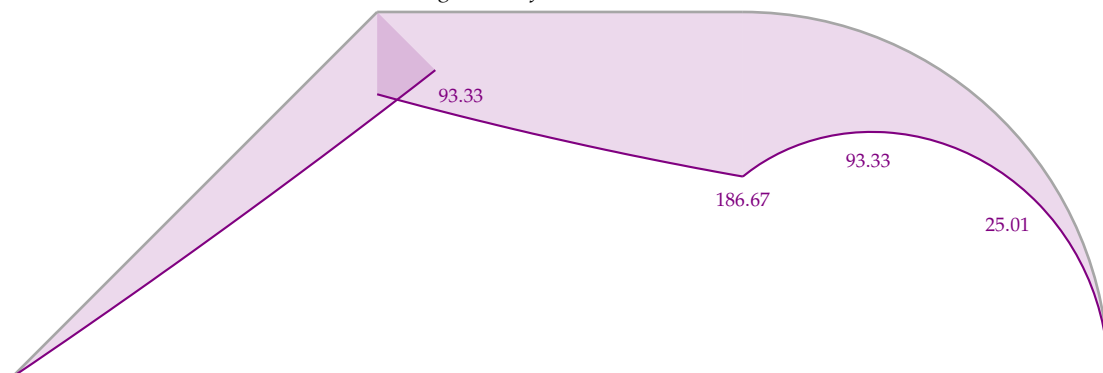
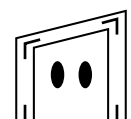


Diagrama de momento flector (M)



Generado con EMI: calculadora estructural online: emiestruc.com