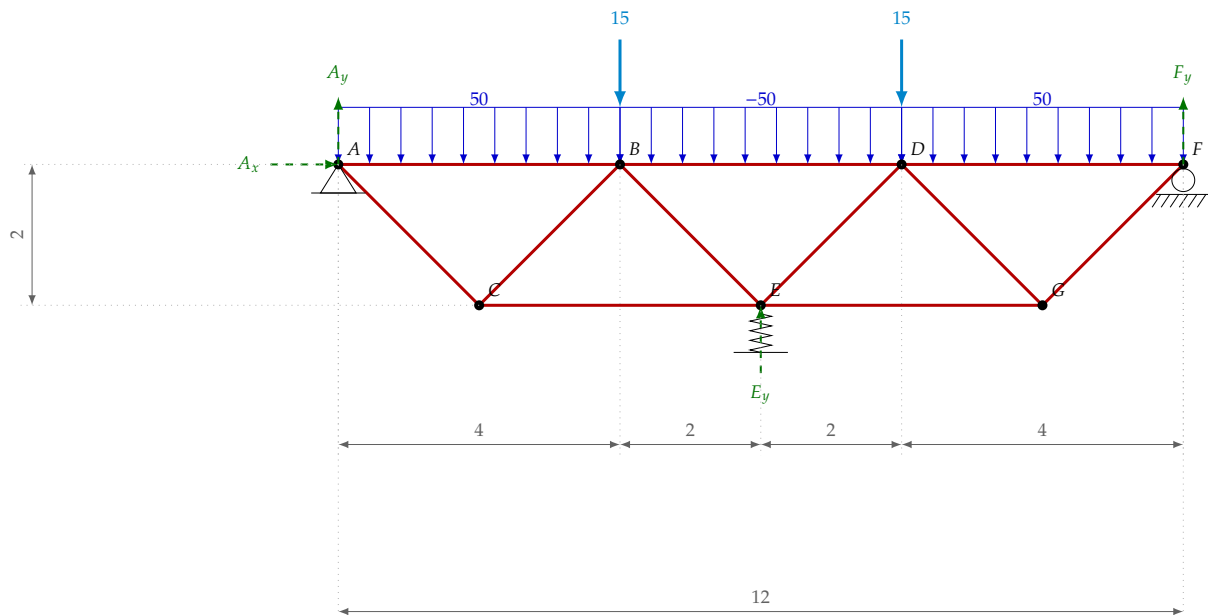


Segundo Teorema de Castigliano

ARMADURA HIPERESTÁTICA

EMI · solución paso a paso



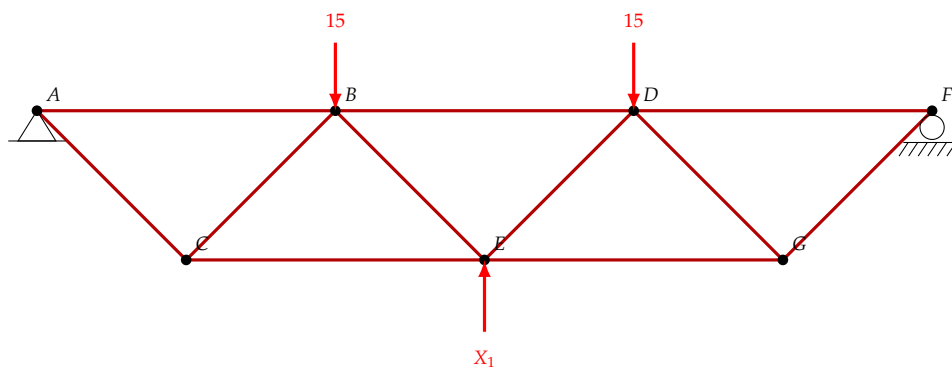
Armatura analizada, con las reacciones por calcular (a trazos, en el sentido positivo supuesto).

Unidades de trabajo: L [m] F [kN] M [kN·m] q [kN/m] δ [mm] E [kN/m²] A [m²] I [m⁴]

1. Clasificación estática (grado total = interno + externo)

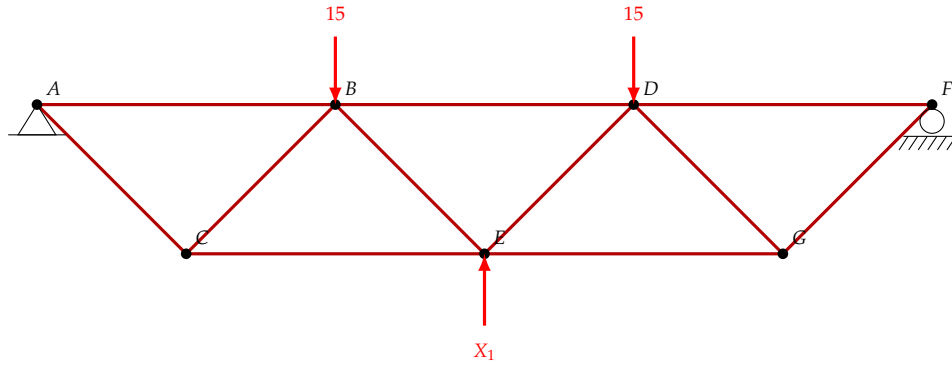
$$GH = m + r - 2n = 11 + 4 - 2(7) = 1$$

2. Redundantes y estructura primaria



Estructura primaria: barras cortadas y reacciones liberadas marcadas con X_i .

3. Estática en función de las redundantes



Estructura con la redundante liberada y aplicada como acción X_1 , junto a las cargas reales.

3.1 Reacciones

$$A_x = 0 \text{ kN}$$

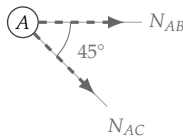
$$A_y = 15 - 0.5 X_1 \text{ kN}$$

$$F_y = 15 - 0.5 X_1 \text{ kN}$$

$$E_y = X_1 \quad (\text{la reacción liberada es la propia redundante})$$

3.2 Equilibrio de nudos

Nudo A



$$(\rightarrow +) \sum F_x = 0 \Rightarrow N_{AB} + N_{AC} \cdot \cos(45^\circ) = 0$$

$$\Rightarrow N_{AB} = -[N_{AC} \cdot \cos(45^\circ)]$$

$$\Rightarrow \boxed{N_{AB} = -15 + 0.5 X_1}$$

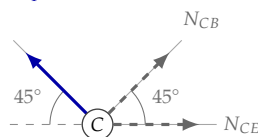
$$(\uparrow +) \sum F_y = 0 \Rightarrow 15 - 0.5 X_1 - N_{AC} \cdot \sin(45^\circ) = 0$$

$$\Rightarrow N_{AC} = \frac{-[15 - 0.5 X_1]}{-\sin(45^\circ)}$$

$$\Rightarrow \boxed{N_{AC} = 21.21 - 0.7071 X_1}$$

Nudo C

$$21.21 - 0.7071 X_1$$



$$(\rightarrow +) \sum F_x = 0 \Rightarrow -(21.21 - 0.7071 X_1) \cdot \cos(45^\circ) + N_{CB} \cdot \cos(45^\circ) + N_{CE} = 0$$

$$\Rightarrow -15 + 0.5 X_1 + N_{CB} \cdot \cos(45^\circ) + N_{CE} = 0$$

$$\Rightarrow N_{CE} = -[-15 + 0.5 X_1 + N_{CB} \cdot \cos(45^\circ)]$$

$$\Rightarrow \boxed{N_{CE} = 30 - X_1}$$

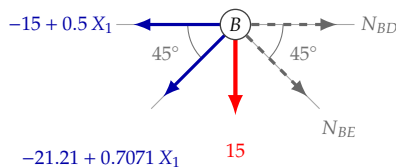
$$(\uparrow +) \sum F_y = 0 \Rightarrow (21.21 - 0.7071 X_1) \cdot \sin(45^\circ) + N_{CB} \cdot \sin(45^\circ) = 0$$

$$\Rightarrow 15 - 0.5 X_1 + N_{CB} \cdot \sin(45^\circ) = 0$$

$$\Rightarrow N_{CB} = \frac{-[15 - 0.5 X_1]}{\sin(45^\circ)}$$

$$\Rightarrow \boxed{N_{CB} = -21.21 + 0.7071 X_1}$$

Nudo B



$$(\rightarrow +) \sum F_x = 0 \Rightarrow -(-15 + 0.5 X_1) - (-21.21 + 0.7071 X_1) \cdot \cos(45^\circ) + N_{BD} + N_{BE} \cdot \cos(45^\circ) = 0$$

$$\Rightarrow 30 - X_1 + N_{BD} + N_{BE} \cdot \cos(45^\circ) = 0$$

$$\Rightarrow N_{BD} = -[30 - X_1 + N_{BE} \cdot \cos(45^\circ)]$$

$$\Rightarrow \boxed{N_{BD} = -30 + 1.5 X_1}$$

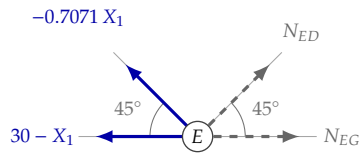
$$(\uparrow +) \sum F_y = 0 \Rightarrow -15 - (-21.21 + 0.7071 X_1) \cdot \sin(45^\circ) - N_{BE} \cdot \sin(45^\circ) = 0$$

$$\Rightarrow -0.5 X_1 - N_{BE} \cdot \sin(45^\circ) = 0$$

$$\Rightarrow N_{BE} = \frac{-[-0.5 X_1]}{-\sin(45^\circ)}$$

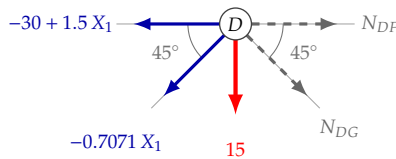
$$\Rightarrow \boxed{N_{BE} = -0.7071 X_1}$$

Nudo E



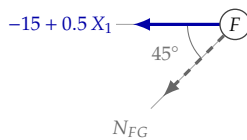
$$\begin{aligned}
 (\rightarrow +) \sum F_x = 0 &\Rightarrow -0.7071 X_1 \cdot \cos(45^\circ) - (30 - X_1) + N_{ED} \cdot \cos(45^\circ) + N_{EG} = 0 \\
 &\Rightarrow -30 + 1.5 X_1 + N_{ED} \cdot \cos(45^\circ) + N_{EG} = 0 \\
 &\Rightarrow N_{EG} = -[-30 + 1.5 X_1 + N_{ED} \cdot \cos(45^\circ)] \\
 &\Rightarrow \boxed{N_{EG} = 30 - X_1} \\
 (\uparrow +) \sum F_y = 0 &\Rightarrow X_1 - 0.7071 X_1 \cdot \sin(45^\circ) + N_{ED} \cdot \sin(45^\circ) = 0 \\
 &\Rightarrow 0.5 X_1 + N_{ED} \cdot \sin(45^\circ) = 0 \\
 &\Rightarrow N_{ED} = \frac{-[0.5 X_1]}{\sin(45^\circ)} \\
 &\Rightarrow \boxed{N_{ED} = -0.7071 X_1}
 \end{aligned}$$

Nudo D



$$\begin{aligned}
 (\rightarrow +) \sum F_x = 0 &\Rightarrow -0.7071 X_1 \cdot \cos(45^\circ) - (-30 + 1.5 X_1) + N_{DG} \cdot \cos(45^\circ) + N_{DF} = 0 \\
 &\Rightarrow 30 - X_1 + N_{DG} \cdot \cos(45^\circ) + N_{DF} = 0 \\
 &\Rightarrow N_{DF} = -[30 - X_1 + N_{DG} \cdot \cos(45^\circ)] \\
 &\Rightarrow \boxed{N_{DF} = -15 + 0.5 X_1} \\
 (\uparrow +) \sum F_y = 0 &\Rightarrow -15 - 0.7071 X_1 \cdot \sin(45^\circ) - N_{DG} \cdot \sin(45^\circ) = 0 \\
 &\Rightarrow -15 + 0.5 X_1 - N_{DG} \cdot \sin(45^\circ) = 0 \\
 &\Rightarrow N_{DG} = \frac{-[-15 + 0.5 X_1]}{-\sin(45^\circ)} \\
 &\Rightarrow \boxed{N_{DG} = -21.21 + 0.7071 X_1}
 \end{aligned}$$

Nudo F



$$\begin{aligned}
 (\rightarrow +) \sum F_x = 0 &\Rightarrow 15 - 0.5 X_1 - N_{FG} \cdot \cos(45^\circ) = 0 \\
 &\Rightarrow N_{FG} = \frac{-[15 - 0.5 X_1]}{-\cos(45^\circ)} \\
 &\Rightarrow \boxed{N_{FG} = 21.21 - 0.7071 X_1} \\
 (\uparrow +) \sum F_y = 0 &\Rightarrow 15 - 0.5 X_1 - N_{FG} \cdot \sin(45^\circ) = 0
 \end{aligned}$$

4. Derivada de los axiles respecto de las redundantes

$$N = N(X_i) \quad \Rightarrow \quad \frac{\partial N}{\partial X_i} = \text{coeficiente de } X_i \text{ en } N$$

5. Segundo teorema de Castigliano

$$\begin{aligned}
 U &= \sum \frac{N^2 L}{2EA} + \sum \frac{X_i^2}{2k} \quad \Rightarrow \quad \frac{\partial U}{\partial X_i} = \sum \frac{N L}{EA} \frac{\partial N}{\partial X_i} + \frac{X_i}{k} \\
 \frac{\partial U}{\partial X_i} &= \sum \frac{N}{EA} \frac{\partial N}{\partial X_i} L + \frac{X_i}{k} = 0
 \end{aligned}$$

Barra	N (kN)	$\frac{\partial N}{\partial X_1}$	E (kN/m ²)	A (m ²)	L (m)	$N \cdot L \cdot \frac{\partial N}{\partial X_1}$
AB	$-15 + 0.5 X_1$	0.5	$2 \cdot 10^8$	0.05	4	$-30 + X_1$
BC	$-21.21 + 0.7071 X_1$	0.7071	$2 \cdot 10^8$	0.05	2.828	$-42.43 + 1.414 X_1$
CA	$21.21 - 0.7071 X_1$	-0.7071	$2 \cdot 10^8$	0.05	2.828	$-42.43 + 1.414 X_1$
DE	$-0.7071 X_1$	-0.7071	$2 \cdot 10^8$	0.05	2.828	$1.414 X_1$
FG	$21.21 - 0.7071 X_1$	-0.7071	$2 \cdot 10^8$	0.05	2.828	$-42.43 + 1.414 X_1$
DB	$-30 + 1.5 X_1$	1.5	$2 \cdot 10^8$	0.05	4	$-180 + 9 X_1$
BE	$-0.7071 X_1$	-0.7071	$2 \cdot 10^8$	0.05	2.828	$1.414 X_1$
EG	$30 - X_1$	-1	$2 \cdot 10^8$	0.05	4	$-120 + 4 X_1$
GD	$-21.21 + 0.7071 X_1$	0.7071	$2 \cdot 10^8$	0.05	2.828	$-42.43 + 1.414 X_1$
DF	$-15 + 0.5 X_1$	0.5	$2 \cdot 10^8$	0.05	4	$-30 + X_1$
CE	$30 - X_1$	-1	$2 \cdot 10^8$	0.05	4	$-120 + 4 X_1$

$$\frac{\partial U}{\partial X_1} = \frac{1}{EA} [-649.7 + 27.49 X_1] + \frac{X_1}{1000} = 0 \quad \Rightarrow \quad -649.7 + 10030 X_1 = 0$$

$$\boxed{X_1 = 0.06479 \text{ kN}}$$

6. Sustitución: reacciones y fuerzas de barra

6.1 En las ecuaciones de equilibrio: reacciones

$$A_x = 0 = 0 \text{ kN}$$

$$A_y = 15 - 0.5 \cdot (0.06479) = 14.97 \text{ kN}$$

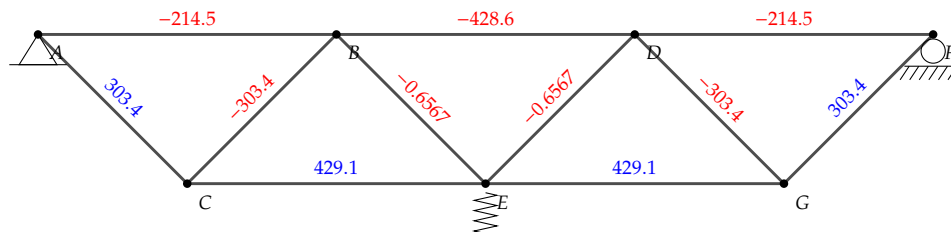
$$F_y = 15 - 0.5 \cdot (0.06479) = 14.97 \text{ kN}$$

$$E_y = X_1 = 0.06479 \text{ kN}$$

6.2 En los axiles: fuerzas de barra

Y la misma X en cada N :

Barra	$N(X)$	N (kN)	Matricial
AB	$-15 + 0.5 \cdot (0.06479)$	-14.97	-214.5
BC	$-21.21 + 0.7071 \cdot (0.06479)$	-21.17	-303.4
CA	$21.21 - 0.7071 \cdot (0.06479)$	21.17	303.4
DE	$-0.7071 \cdot (0.06479)$	-0.04582	-0.6567
FG	$21.21 - 0.7071 \cdot (0.06479)$	21.17	303.4
DB	$-30 + 1.5 \cdot (0.06479)$	-29.9	-428.6
BE	$-0.7071 \cdot (0.06479)$	-0.04582	-0.6567
EG	$30 - (0.06479)$	29.94	429.1
GD	$-21.21 + 0.7071 \cdot (0.06479)$	-21.17	-303.4
DF	$-15 + 0.5 \cdot (0.06479)$	-14.97	-214.5
CE	$30 - (0.06479)$	29.94	429.1



Esfuerzos axiales sobre las barras (kN): en azul las de tracción (+) y en rojo las de compresión (-).

Diagramas de fuerzas internas

Esfuerzo normal N , constante a lo largo de cada barra

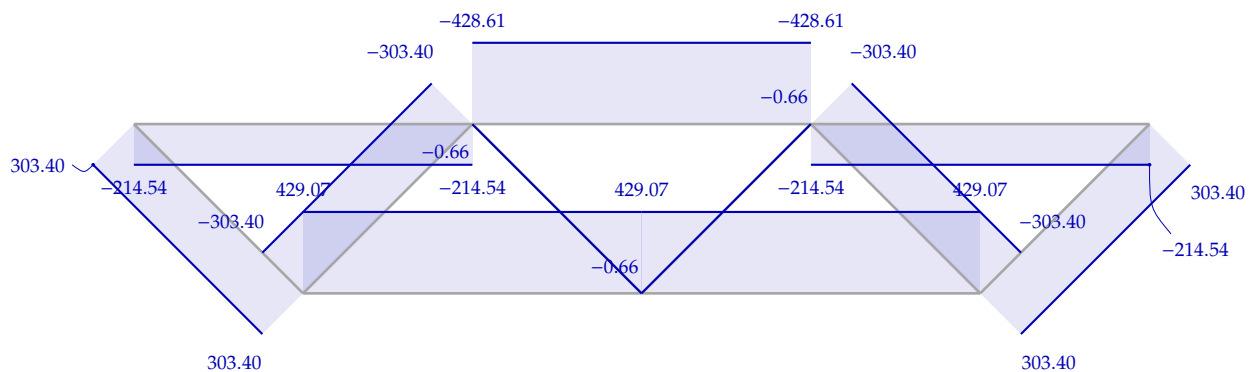
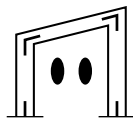


Diagrama de fuerza axial (N)



Generado con EMI: calculadora estructural online: emiestruc.com